

# Anisotropic renormalized fluctuations in the microwave resistivity in $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$

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We discuss the excess conductivity above  $T_c$  due to renormalized order-parameter fluctuations in  $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$  (YBCO) at microwave frequencies. We calculate the effects of the uniaxial anisotropy on the renormalized fluctuations in the Hartree approximation, extending the isotropic theory developed by Dorsey [*Phys. Rev. B* **43**, 7575 (1991)]. Measurements of the real part of the microwave resistivity at 24 and 48 GHz and of the dc resistivity are performed on different YBCO films. The onset of the superconducting transition and the deviation from the linear temperature behavior above  $T_c$  can be fully accounted for by the extended theory. According to the theoretical calculation here presented, a departure from gaussian toward renormalized fluctuations is observed. Very consistent values of the fundamental parameters (critical temperature, coherence lengths, penetration depth) of the superconducting state are obtained.

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## I. INTRODUCTION

The analysis of fluctuations-induced excess conductivity has stimulated in the past years a considerable amount of work. Theoretical investigations of the dc as well as the finite-frequency conductivity dates from the 1960's,<sup>1–5</sup> and development of this topic proceeded until the discovery of high temperature superconductors (HTSC's). High critical temperatures and short coherence lengths conspire to the giant enhancement of thermodynamical fluctuations in HTCs. Due to their layered structure and to the consequent anisotropy in the superconducting state, the effects of thermal fluctuations are further enhanced. As a result, these materials can be viewed as ideal systems to experimentally verify the theories for the excess conductivity. Among the high-temperature compounds with moderate but significant anisotropy, YBCO is the most studied on this aspect,<sup>6–15</sup> and we will restrict our discussion and measurements to this compound only.

Despite the extended experimental investigation, there is a considerable debate on the appropriate model for the fluctuation conductivity in zero magnetic field:<sup>7,8,15–17</sup> most of the existing analysis of the effects of order-parameter fluctuations on the dc conductivity have been performed in terms of Aslamazov-Larkin (AL) isotropic three-dimensional (3D) fluctuations<sup>2</sup> for temperatures close to  $T_c$ , while a crossover to 2D fluctuations has been claimed at temperatures substantially higher than  $T_c$ ,<sup>7</sup> according to the interpretation of the data in terms of Lawrence-Doniach (LD) model.<sup>18</sup> Not too close to  $T_c$ , data on YBCO have been shown to be compatible with an AL interpretation supplemented with a Maki-Thompson

(MT) term.<sup>5,7</sup> However this framework has been seriously questioned by Hopfgartner *et al.*<sup>8</sup> by extending the AL theory to an anisotropic superconductor and introducing a phenomenological cutoff for long wave-vector fluctuations they showed that the dc excess conductivity in YBCO films agreed well with the modified AL expression, without the need for MT terms (up to  $T = 1.1 T_c$ ). Most important, from this kind of analysis no 3D-2D crossover was found in YBCO, in contrast with the ordinary LD-like crossover.<sup>8</sup>

These interpretations are based on theoretical results obtained in the Gaussian approximation. Approaching the critical region close to  $T_c$  this treatment must be extended to take into account interactions between fluctuations. The amplitude of the critical region is theoretically predicted to be experimentally accessible for these superconductors,<sup>19</sup> but the actual value of the crossover temperature from Gaussian to critical behavior is still debated.<sup>15,20</sup>

Up to now, the analysis of the dynamical properties near  $T_c$  have been performed mainly through the comparison of experimental data with appropriate power laws of the reduced temperature. However, experimental data give controversial results. For example, the dc conductivity above  $T_c$ <sup>21</sup> and the penetration depth  $\lambda(T)$  below  $T_c$ <sup>11</sup> have been found to follow a 3D XY-like power law, but Gaussian results for  $\lambda(T)$  have been recently reported.<sup>12</sup> Moreover, a possible crossover from critical to 2D Gaussian fluctuations has been reported in the microwave conductivity.<sup>13</sup> In fact, simple power laws or scaling behaviors, without explicit expressions for the various quantities, do not allow a quantitative and complete comparison between experiments and theories.

Explicit expressions for the finite-frequency conductivity have been recently calculated by Dorsey<sup>22</sup> beyond the Gaussian approximation, using a Hartree approach; in this treatment, confined to an isotropic, three dimensional superconductor, a renormalized expression for the 3D isotropic fluctuational conductivity is deduced, explicitly as well as in a scaling form. The basic scaling parameter is the temperature-dependent correlation time  $\tau \sim \xi^z$ , with the correlation length diverging at  $T = T_c$  as  $\xi \sim \epsilon^{-\nu}$ , where sufficiently close to  $T_c$ ,  $\epsilon = (T/T_c - 1)$ . Measurements of the complex conductivity as a function of frequency<sup>9</sup> analyzed in terms of the abovementioned theory have revealed a somehow puzzling behavior: in fact, the complex conductivity  $\sigma(\omega)$  does exhibit a scaling behavior close to the expected one, but the so-obtained critical exponents,  $\nu \simeq 1.2$  and  $z \simeq 2.6$ , are quite different with respect to the Gaussian values,  $\nu = 0.5$  and  $z = 2$ ; the critical exponent  $\nu$  is also in conflict with the prediction for the 3D XY uncharged fluid,  $\nu = 2/3$ .<sup>23</sup> The determination of the critical exponents close to  $T_c$  is uncertain: in fact, a different scaling analysis of measurements of the frequency-dependent conductivity up to 2 GHz in zero magnetic field gave large exponents,  $\nu \simeq 1.7$  and  $z \simeq 5.6$ ,<sup>10</sup> in contrast with those previously obtained.

A noticeable fact in the existing body of experimental data and theoretical models is that the commonly performed analyses do not explicitly include the anisotropy, so that an intrinsic feature of HTSC's is lost. In particular, while there are Gaussian theories for the anisotropic fluctuational conductivity,<sup>4</sup> the inclusion of at least a mass tensor in a renormalized theory for the finite-frequency fluctuational conductivity is still missing, at least to our knowledge. Since in materials such as

HTSC's the intrinsic anisotropy is one ingredient that possibly makes the departure from Gaussian behavior experimentally observable, we think that a quantitative analysis of the data must be based on the explicit inclusion of the anisotropy in the calculations.

In this paper we extend the renormalized-fluctuations theory developed by Dorsey<sup>22</sup> by introducing an anisotropic mass tensor, and we compare the results to resistive transitions in zero field obtained in dc and at high frequency, above the critical temperature. In Section II, we calculate the Gaussian and renormalized-fluctuations-induced excess conductivity above  $T_c$  in a uniaxial superconductor, subjected to an alternating electric field along the  $(a, b)$  planes, stressing the main differences that come out by the introduction of the anisotropy. We write down the explicit expressions for the dynamical conductivity as a function of the frequency and temperature, in terms of physical parameters (coherence lengths, penetration depth). In Section III we briefly describe the samples under study, we sketch the experimental apparatuses and we present the resistive transitions in dc, at 24 and 48 GHz. In Section IV we show that very good fits to the data are obtained with the extended theory here developed, with very reasonable parameters. A smooth departure from Gaussian fluctuations is obtained.

## II. THEORY

To take into account the intrinsic anisotropy of HTSC's, we start from the standard Ginzburg-Landau functional for a uniaxial anisotropic superconductor in presence of an external potential vector  $\mathbf{A}$  (throughout the paper we use Système International units):

$$F = \int d\mathbf{r} \left[ \sum_{j=x,y,z} \frac{\hbar^2}{2m_j} \left| \left( \frac{\partial}{\partial r_j} - \frac{ie^*}{\hbar} A_j \right) \psi(\mathbf{r}) \right|^2 + \alpha |\psi(\mathbf{r})|^2 + \frac{1}{2} \beta |\psi(\mathbf{r})|^4 \right] \quad (1)$$

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where  $e^* = 2e$  is twice the electronic charge,  $m_{x,y} = m_{ab}$  and  $m_z = m_c$  are the masses of the pair along the main crystallographic directions, the coefficient  $\alpha$  is a linear function of the reduced temperature  $\alpha = a\epsilon$ , and  $\epsilon = \ln(T/T_c)$  is the reduced temperature.<sup>24</sup> Our aim is to calculate the dynamical conductivity along the  $(a, b)$

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planes (e.g. the  $x$  axis) in presence of an electric field  $\mathbf{E}$ , or equivalently, a vector potential  $\mathbf{A}$ . In order to calculate a dynamical property of the system, such as the conductivity, we consider the temporal evolution as determined by the time dependent Ginzburg-Landau equation

$$\begin{aligned} & \frac{1}{\Gamma_0} \left( \frac{\partial}{\partial t} + \frac{ie^*}{\hbar} \phi(\mathbf{r}, t) \right) \psi(\mathbf{r}, t) = -\frac{\delta F[\psi]}{\delta \psi^*} + \zeta(\mathbf{r}, t) = \\ & = - \left[ \alpha + \beta |\psi|^2 - \frac{\hbar^2}{2m_{ab}} \left( \frac{\partial}{\partial x} - \frac{ie^*}{\hbar} A_x \right)^2 - \frac{\hbar^2}{2m_{ab}} \frac{\partial^2}{\partial y^2} - \frac{\hbar^2}{2m_c} \frac{\partial^2}{\partial z^2} \right] \psi(\mathbf{r}, t) + \zeta(\mathbf{r}, t) \end{aligned} \quad (2)$$


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where  $\Gamma_0$  is a constant relaxation time; thermal fluctuations are represented by the noise term  $\zeta(\mathbf{r}, t)$  with  $\delta$  function correlation  $\langle \zeta^*(\mathbf{r}, t) \zeta(\mathbf{r}', t') \rangle = (2k_B T / \Gamma_0) \delta(\mathbf{r} - \mathbf{r}') \delta(t - t')$ . We choose the gauge where the scalar potential  $\phi(\mathbf{r}, t) = 0$ .

The calculation scheme is as follows: we first compute the conductivity above  $T_c$  in the linear response in the Gaussian approximation. The result will depend on the temperature through  $\alpha(T)$ . Then we renormalize the  $\alpha$  parameter by using an Hartree approximation for the quartic term in the GL functional. Inserting the latter in the Gaussian conductivity we get the renormalized result. This approach has been used in Ref. 22 to calculate the linear and nonlinear excess conductivity in an isotropic superconductor. While our analysis explicitly include the anisotropy in the calculation of the linear conductivity,

nonlinear effects are beyond the purposes of this paper. The response of the system to the in-plane field  $\mathbf{A}(t)$  is determined by the current operator averaged with respect to the noise (here represented by the brackets): it can be expressed as a function of the correlation function of the order parameter  $C(\mathbf{r}, t; \mathbf{r}', t') = \langle \psi(\mathbf{r}, t) \psi^*(\mathbf{r}', t') \rangle$ :

$$\langle J_x(t) \rangle = -\frac{\hbar e^*}{m_{ab}} \int \frac{d^3 \mathbf{q}}{(2\pi)^3} q_x C \left[ \mathbf{k} = \mathbf{q} - \frac{e^*}{\hbar} \mathbf{A}(t); t, t \right] \quad (3)$$

where the momentum dependence has been shifted from  $\mathbf{k}$  to the new vector  $\mathbf{q} = \mathbf{k} + (e^*/\hbar) \mathbf{A}(t)$ . As a first step (Gaussian approximation), we neglect the non linear term  $\beta |\psi|^2 \psi$ . Eq.1 is then exactly solvable, and for the correlation function one gets:

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$$C(\mathbf{q}; t, t) = 2k_B T \Gamma_0 \int_0^{+\infty} \exp \left\{ -2\Gamma_0 \alpha s - \Gamma_0 \hbar^2 s \left( \frac{q_y^2}{m_{ab}} + \frac{q_z^2}{m_c} \right) + \right. \\ \left. - \frac{\Gamma_0 \hbar^2 s}{m_{ab}} \left[ q_x + \frac{e^*}{\hbar s} \int_0^s du [A_x(t-u) - A_x(t)] \right]^2 - \frac{\Gamma_0 e^{*2}}{m_{ab}} \left[ \int_0^s du (A_x(t-u))^2 - \frac{1}{s} \left( \int_0^s du A_x(t-u) \right)^2 \right] \right\} ds \quad (4)$$


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In the frame of the linear response, the quadratic terms in the vector potential can be neglected. Using the ex-

pression found for the correlation function, Eq.4, the current operator in Eq.3 becomes

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$$\langle J_x(t) \rangle = \frac{e^2}{\hbar^3} k_B T \left( \frac{m_c}{\Gamma_0 \pi^3} \right)^{1/2} \int_0^{+\infty} ds \left[ \frac{e^{-2\Gamma_0 \alpha s}}{s^{5/2}} \int_0^s du [A_x(t-u) - A_x(t)] \right] \quad (5)$$


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After Fourier transformation, one has  $J_x = [\sigma'(\omega) + i\sigma''(\omega)] E_x$ , with

$$\sigma'(\omega) = \frac{e^2}{\hbar^3} k_B T \left( \frac{m_c}{\Gamma_0 \pi^3} \right)^{1/2} \int_0^{+\infty} \frac{ds}{s^{5/2}} e^{-2\Gamma_0 \alpha s} \frac{1 - \cos \omega s}{\omega^2} \quad (6)$$

$$\sigma''(\omega) = \frac{e^2}{\hbar^3} k_B T \left( \frac{m_c}{\Gamma_0 \pi^3} \right)^{1/2} \int_0^{+\infty} \frac{ds}{s^{5/2}} e^{-2\Gamma_0 \alpha s} \frac{\omega s - \sin \omega s}{\omega^2} \quad (7)$$

After integration, Eqs.6 and 7 can be written as:

$$\sigma_g(\omega) = \sigma'(\omega) + i\sigma''(\omega) = \\ = \frac{e^2}{32\hbar \xi_{c0}} \frac{1}{\epsilon^{1/2}} \left[ S_+ \left( \frac{\omega}{\Omega} \right) + i S_- \left( \frac{\omega}{\Omega} \right) \right] \quad (8)$$

where  $S_+(x)$  and  $S_-(x)$  are the scaling functions as can be found in Ref. 22 (the subscript  $g$  means that this result is obtained in the Gaussian approximation), and they have the property that  $S_+(x \rightarrow 0) = 1$  and  $S_-(x \rightarrow 0) = 0$ . The characteristic frequency  $\Omega$  is

$$\Omega = 2\Gamma_0 \alpha = \frac{32k_B T}{\hbar} \epsilon \quad (9)$$

where the relaxation time  $\Gamma_0 = (8k_B T / \hbar \pi a)$  is evaluated from the microscopic theory<sup>1</sup> and  $\xi_{c0} = \hbar / (2m_c a)^{1/2}$  is the zero temperature c-axis correlation length. Before proceeding further to the renormalization, some comments are in order. Eq.8 contains all the previously obtained results in various limits: as  $\omega \rightarrow 0$  Eq.8 gives the

dc, anisotropic AL result.<sup>8,25</sup> As expected, at nonzero frequencies the conductivity does not diverge at  $T_c$ , due to the vanishing of the scaling functions  $S_{\pm}$  when written in terms of the temperature. Moreover, the result in Eq.8 agrees with the one calculated by a different approach by Klemm.<sup>4</sup> The isotropic result is recovered by simply putting  $\xi_{c0} = \xi$ . The introduction of the anisotropy leads to an enhancement by a factor of  $\gamma = \xi_{ab0}/\xi_{c0}$  in the prefactor of the fluctuation conductivity, as seen by the fact that Eq.8 contains only the short coherence length  $\xi_{c0}$ . However, *in the Gaussian approximation*, the characteristic frequency remains unchanged with respect to the isotropic result. This is no longer true in the renormalized regime, as we show below.

Approaching  $T_c$ , the Gaussian approximation breaks down. We extend our calculation to this region by considering the effects of the interaction term  $\beta|\psi|^2\psi$  of the Landau-Ginzburg functional through the Hartree approximation: we replace the non-linear term by its average value and put it in a renormalization of the parameter  $\alpha$ . The renormalized parameter  $\tilde{\alpha}$

$$\tilde{\alpha} = \alpha + \beta \left\langle |\psi|^2 \right\rangle = \alpha + \beta \int \frac{d^3\mathbf{q}}{(2\pi)^3} C(\mathbf{q}; t, t) \quad (10)$$

represents the renormalized temperature dependence and it is defined through this self-consistency equation. All the quantities above calculated in the Gaussian approximation contain the temperature dependence through the parameter  $\alpha$ ; hence, the relations found can be easily extended to the critical region by replacing  $\alpha \rightarrow \tilde{\alpha}$ . In particular, the correlation function is formally identical to the one determined by means of Eq.4; evaluating it with an electric field along the  $x$ -axis in the frame of the linear response approximation, Eq.10 becomes:

$$2m_{ab}\tilde{\alpha} = 2m_{ab}(\alpha - \alpha_c) - \frac{\beta k_B T}{\hbar^3 \pi} (m_{ab}^3 m_c)^{1/2} (2m_{ab}\tilde{\alpha})^{1/2} \quad (11)$$

$$\tilde{\sigma}(\omega, T) = \frac{e^2}{32\hbar} \gamma \frac{\tilde{\xi}_{ab}(T)}{\xi_{ab0}^2} \left[ S_+ \left( \frac{\omega}{\tilde{\Omega}(T)} \right) + iS_- \left( \frac{\omega}{\tilde{\Omega}(T)} \right) \right] = \tilde{\sigma}_{dc}(T) [S_+ + iS_-] \quad (13)$$

where

$$\tilde{\Omega}(T) = \frac{32k_B T}{h} \left( \frac{\xi_{ab0}}{\tilde{\xi}_{ab}(T)} \right)^2 \quad (14)$$

and  $\tilde{\tau} = 1/\tilde{\Omega}(T)$  plays the role of the renormalized scattering time. Expression (13) takes then the form of the renormalized dc excess conductivity  $\tilde{\sigma}_{dc}(T)$  times a frequency-dependent contribution. As it can be noted from Eq.12, the expression for the fluctuation conductivity depends on a limited number of parameters, namely the bare quantities  $\xi_{ab0}$ ,  $\gamma$  and  $\kappa$ , being  $\tilde{T}_c$  the only renormalized parameter. We note that now the (renormalized)

where  $\alpha_c$  is the bare  $\alpha$  parameter evaluated at the renormalized critical temperature  $\tilde{T}_c$  at which the parameter  $\tilde{\alpha}$  vanishes:  $\alpha_c = \alpha(T = \tilde{T}_c) = a \ln(\tilde{T}_c/T_c)$ . The self consistency equation, Eq.11, can be usefully interpreted as the relation which determines the renormalized correlation length along the  $(a, b)$  planes  $\tilde{\xi}_{ab} = (\hbar^2/2m_{ab}\tilde{\alpha})^{1/2}$ :

$$\tilde{\xi}_{ab}(T) = \frac{w \kappa^2 \gamma \xi_{ab0}^2}{\ln(T/\tilde{T}_c)} \left[ 1 + \left( 1 + \frac{\ln(T/\tilde{T}_c)}{w^2 \kappa^4 \gamma^2 \xi_{ab0}^2} \right)^{1/2} \right] \quad (12)$$

where  $w = (e^2 \mu_0 k_B T / \pi \hbar^2)$ ,  $\mu_0$  is the magnetic permeability of vacuum,  $\gamma = (\xi_{ab0}/\xi_{c0})$  is the anisotropy factor and  $\kappa = (\lambda_{ab0}/\xi_{ab0})$  is the Ginzburg parameter, which is related to the coefficient  $\beta$  of the Landau-Ginzburg functional through the London equation.<sup>26</sup> The renormalized coherence length follows the usual, Gaussian temperature behavior  $\epsilon^{-\nu}$  with  $\nu = 1/2$  sufficiently far away from  $T_c$ , but approaching the critical temperature it diverges with the critical exponent  $\nu = 1$ , as depicted in Fig.1. A smooth crossover between these two power laws is then obtained by varying the temperature (Fig.1). It is interesting to note that, while the exponent  $\nu = 1$  is recovered only very close to  $T_c$ , a substantial departure from the Gaussian value  $\nu = 1/2$  is obtained at rather high temperatures, and this regime is very well approximated by an exponent  $\nu = 2/3$ . Once the renormalized coherence length is obtained, the conductivity can be immediately written down by substituting  $\xi_{ab}(T)$  with  $\tilde{\xi}_{ab}(T)$ , and one gets

scattering time *does* depend on the anisotropy, differently from the Gaussian result, Eq.9. As expected, in the limit  $\gamma = 1$  our results coincide with the isotropic calculation.<sup>22</sup> We stress, however, that the anisotropy  $\gamma$  does not enter in a trivial way in the renormalized quantities: it enters through different combinations in the prefactor of  $\tilde{\sigma}$  and in the renormalized scattering time. In particular, our calculation cannot be simply mapped onto the isotropic result by the use of some "lumped" parameter:  $\gamma$  is an *independent* parameter.

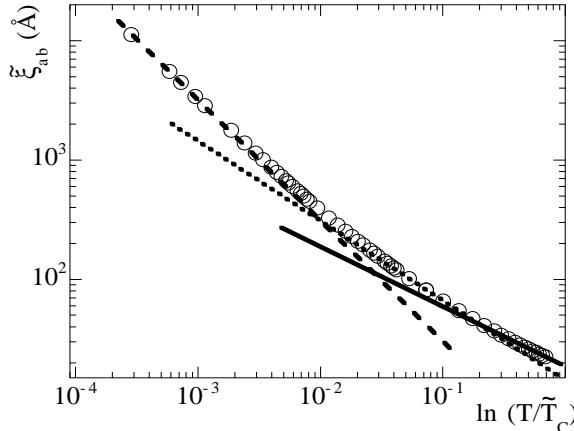


FIG. 1. Renormalized coherence length as a function of the temperature,  $\xi_{ab}$  (open circles). Parameters appropriate to the fitting of the data on sample IV were used. Various power laws are depicted, in the form  $\xi_{ab} \sim [(\ln(T/\tilde{T}_c))]^{-\nu}$ : Gaussian ( $\nu = 1/2$ , full line), critical ( $\nu = 1$ , dashed line) and  $\nu = 2/3$  (dotted line).

### III. EXPERIMENTAL SECTION

Measurements of the microwave and dc resistivity in nominally zero field were performed. Five YBCO thin films, grown by different methods<sup>27–29</sup> were investigated. All samples were highly c-axis oriented, as indicated from the  $\theta - 2\theta$  rocking curve. Twinning is largely present in all samples (as usual in films). Thicknesses ranged from 0.08 to 0.5  $\mu\text{m}$ . The main features are presented in Table I. It is worth mentioning that samples II and III are of inferior quality, as can be seen, e.g., by the fact that the normal state resistivity is 2-3 times higher than in the other films.

Microwave resistivity measurements were performed on as-deposited samples I-IV. The microwave response was investigated at 48 and 24 GHz, in samples I-III and IV, respectively. Extensive descriptions of the experimental systems have been given previously,<sup>30,31</sup> and we give here only a short sketch. Two experimental systems were employed. In both cases we made use of the cavity-end-

wall-replacement method: the sample is mounted in order to replace one end-wall of a mechanically tunable right-cylinder resonant cavity. The cavities were designed to work in absorption in the TE<sub>011</sub> mode, at 24 [cavity (a)] and 48.2 GHz [cavity (b)], with quality factors of about 15000 and 6000, respectively. The relatively low quality factor prevented an accurate measurement of the absolute surface resistance below  $\sim 70\text{K}$ , but allowed us to obtain reliable measurements in the whole transition range and well above  $T_c$ . As described in Ref. 31, the unloaded quality factor  $Q$  of the cavities was measured by recording the (Lorentzian) resonance shape as a function of the slowly ( $\sim 0.2\text{ K/min}$ ) increasing temperature. Changes of  $Q$  reflect the changes in the microwave surface resistance.<sup>31</sup> A calibration of the cavity response is in principle needed to obtain the absolute surface resistance, but since below  $\sim 70\text{ K}$  the changes in  $Q$  cannot be resolved due to the reduced sensitivity of the cavities, data presented as:

$$R_S(T) - R_S(70\text{K}) = G \left[ \frac{1}{Q(T)} - \frac{1}{Q(70\text{K})} \right] \quad (15)$$

are independent on the calibration (a detailed discussion can be found in Ref. 31). This is equivalent to take as zero the low-temperature value of  $R_S$ . Here  $G$  is a known geometrical factor.

The real part of the microwave resistivity is directly obtained from the data due to the reduced sample thickness  $d$ . For samples thinner than twice the penetration depth ( $\delta$  in the normal state, the London penetration depth  $\lambda$  in the superconducting state) the measured surface resistance directly gives the real part of the resistivity through:

$$\text{Re}[\rho(T)] \cong R_S(T) d \quad (16)$$

A detailed study of the applicability range of this approximation can be found in Ref. 32. We notice, however, that we are interested in measurements close and above  $T_c$ , where  $\lambda$  is much longer than the zero-temperature value. Consequently, Eq. 16 is valid up to (at least) 3%,<sup>32</sup> also in the thicker film.

Sample	Thickness( $\mu\text{m}$ )	Substrate	$\omega$ (GHz)	$T_i$	$\tilde{T}_c$	$\xi_{ab0}$ (Å)	$\lambda_{ab0}$ (Å)	$\gamma$
I	0.3	LaAlO <sub>3</sub>	48.2	86.2	86.7	14.4	1300	6.4
II	0.08	LaAlO <sub>3</sub>	48.2	86.5	86.8	17.5	1300	5
III	0.5	LaAlO <sub>3</sub>	48.2	87.2	87.2	16.5	1500	5.4
IV	0.12	LaAlO <sub>3</sub>	24.0	84.6	84.4	14.3	1500	6.6
V	0.1	SrTiO <sub>3</sub>	dc	89.7	89.6	15.0	1100	6.3

TABLE I. Sample characteristics, measuring frequency ( $\omega$ ), measured inflection temperature ( $T_i$ ) and fit parameters ( $\tilde{T}_c$ ,  $\xi_{ab0}$ ,  $\lambda_{ab0}$ ,  $\gamma$ ).

The data for the so-obtained microwave resistivity on four samples as a function of the temperature are reported in Figs.2 and 3 in terms of  $\Delta\rho(T) = \rho(T) - \rho(70K)$ , accordingly to Eqs.15 and 16 . The behavior is essentially linear at high temperatures, and gradually bends approaching the transition. The inflection temperatures of the microwave resistivity,  $T_i$ , in the various samples are reported in Table I. Sample IV was measured at 24 GHz, and samples I-III at 48.2 GHz. Additionally, we performed dc resistivity measurements on a patterned sample. Sample V was patterned in a 2 mm long, 100  $\mu\text{m}$  wide strip by ion milling. The dc resistivity was measured through a four-probe lock-in method, with the current oscillating at 20 Hz. Low current density ( $\sim 10 \text{ A/cm}^2$ ) was used. Data were collected upon cooling and warming in a commercial helium flow cryostat. No differences were observed in the corresponding resistive transition, within the voltage ( $\sim 5 \text{ nV}$ ) and temperature ( $\sim 0.005 \text{ K}$ ) sensitivities. Zero resistance (within our resolution) was attained at  $T_z=88.8 \text{ K}$ .

#### IV. DISCUSSION AND CONCLUSIONS

We proceed here to the fitting of the zero-field resistive transitions above  $T_c$  on the five samples with the proposed model. First of all, we write the total conductivity as the sum of the normal and fluctuational terms, so that the total resistivity is given by

$$\rho(T, \omega) = \frac{1}{\sigma_n(T) + \tilde{\sigma}_{fl}(T, \omega)} = \frac{1}{\frac{1}{\rho_n(T)} + \tilde{\sigma}_{fl}(T, \omega)} \quad (17)$$

where we have assumed that the normal state relaxation time is much smaller than  $1/\omega$ . Here,  $\tilde{\sigma}_{fl}(T, \omega)$  is the fluctuational conductivity and  $\rho_n(T)$  is the measured normal-state resistivity. The latter is linear above  $\sim 120 \text{ K}$ , and it was linearly extrapolated down to  $T_c$ .

Fits of the experimental data with the real part of Eq.17 can now be made. The theoretical expression depends on four independent parameters, that can be chosen to be the bare  $\xi_{ab0}$ ,  $\lambda_{ab0}$  and  $\gamma$ , and the renormalized critical temperature  $T_c$  (which is, in this frame, the experimentally observable critical temperature). With the present interpretation, no information comes from the data for the bare critical temperature,  $T_c$ .

Before commenting on the fits, a few notes should be added. First of all, as common to many calculations of the fluctuational conductivity,<sup>14</sup> the theory overestimates the fluctuational contribution for  $T \gg T_c$ . This is true also at zero frequency. In fact, calculations of the dc excess conductivity in the Gaussian approximation, explicitly including a high- $\mathbf{q}$  cutoff in the various integrations in the momentum space show a drastic and sharp suppression of  $\tilde{\sigma}(T, \omega = 0)$  at sufficiently high  $T$  (above  $\sim 120 \text{ K}$  for YBCO).<sup>8,15,16</sup>

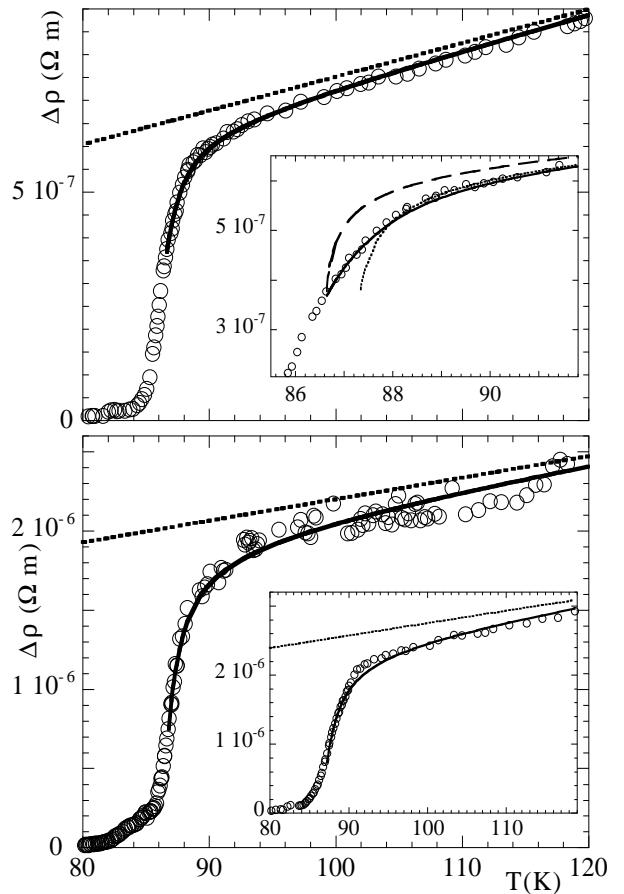


FIG. 2. Measurements of the microwave resistivity at 48.2 GHz on sputtered samples I-III (open circles) and fits through Eq.17 (continuous lines). Normal state resistivity: dotted lines. Upper panel: data for the optimized sample (sample I), with lower normal state resistivity. In the inset: enlargement in a restricted temperature range. The anisotropic Gaussian fits through Eq.8 are also reported for comparison: dashed line (same parameters as for the renormalized fit) and dotted line ( $T_c=87.3 \text{ K}$ ,  $\xi_c = 1.7 \text{\AA}$ ). The Gaussian fits do not reproduce the shape of the transition. Lower panel: data for lower quality samples II (main panel) and III (inset). Scattering of the data above  $T_c$  are due to the sensitivity limit of the cavity.

Inclusion of such a cutoff in the frequency-dependent fluctuational conductivity is beyond of the scope of this paper. Instead, we will phenomenologically take into account this effect by making the physically reasonable assumption that at sufficiently high temperature (say, 150 K) the fluctuational conductivity is no longer resolved in the measurements. As a consequence, we will write the dc excess conductivity in Eq.13 (and then in Eq.17) as  $\tilde{\sigma}_{dc}(T) - \tilde{\sigma}_{dc}(150 \text{ K})$ . Other possible choices consist essentially of taking a normal-state resistivity which is higher than the measured one<sup>14</sup> or with a different shape with respect to the linear extrapolation.<sup>7</sup> However, our present choice keeps as much contact as possible with the measured data well above  $T_c$ , and does not introduce additional parameters.

A second point comes from the fact that the HF renormalization here presented is performed in the adiabatic limit, that is  $\omega/\tilde{\Omega}(T) \ll 1$ . As a consequence, we expect that  $\omega/\tilde{\Omega}(T) \approx 1$  is a limit for the accurate applicability of such calculations, while the main features might apply also beyond this point until the HF approach breaks down very near  $T_c$ . Since  $\tilde{\Omega}(T \rightarrow T_c) \rightarrow 0$ , all the fits were performed self-consistently, excluding the data points with temperatures lower than those for which  $\omega/\tilde{\Omega}(T) = 1$ . This requirement, together with the expected cutoff at high temperature, resulted in fits being performed using the data from about 0.5 K above the critical temperature, up to  $\sim 120$  K.

With these cares, we performed the fits with the fluctuational conductivity here calculated. The results are reported in Figs.2,3 and 4. As can be seen, all the fits are very satisfying, and the anisotropic renormalized fluctuational conductivity seems to be a good description of the data. All the parameters obtained are in the range of commonly reported values (see Table I).<sup>6,33–38</sup> In particular, we note that the low-quality films (samples II and III) have lower  $\gamma$ , as expected and usually found in literature. As a matter of fact, all the  $\tilde{T}_c$ 's almost coincide with the inflection of the transition, well below the first onset of the superconductivity. If one allows the bare critical temperature to be at the onset of superconductivity, one has a rough estimate  $|\tilde{T}_c - T_c| \sim 3$  K.

While the condition  $\omega/\tilde{\Omega}(T) = 1$  is reached about  $\sim 0.5$  K above  $\tilde{T}_c$  (slightly depending on the sample), we note that the fits are in fairly good agreement with the data even down to  $T_c$ . It should be mentioned that preliminary analysis of the flux-flow resistivity<sup>39</sup> at 48GHz below  $T_c$  on sample I gave an independent estimate for the coherence length, in agreement with the value here obtained from zero-field fluctuational conductivity *above*  $T_c$ .

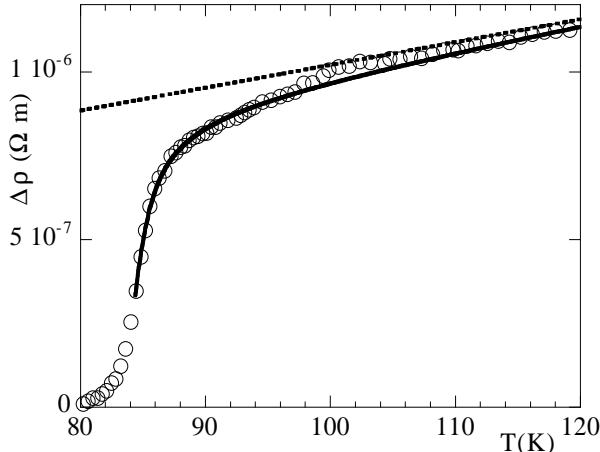


FIG. 3. As in Fig.2, for the microwave resistivity at 24 GHz on the laser-ablated sample IV.

We mention that our data might be equally well fitted with the original isotropic theory<sup>22</sup>. However in this case the parameters attain unrealistic values. As an example, on sample I one would get a fit with the isotropic theory almost indistinguishable from the one obtained with the anisotropic theory with  $\xi = 2.6\text{\AA}$ . Since here  $\xi$  is an isotropic coherence length, it is reasonable to assume  $\xi = (\xi_{ab}^2 \xi_c)^{1/3} = \xi_{ab}/\gamma^{1/3}$ . For  $\xi_{ab} \approx 15\text{\AA}$  one would get  $\gamma \approx 200$ , more than an order of magnitude higher than known values in YBCO. The Gaussian anisotropic theory (Eq.8) can be made to fit the data only in the high part of the resistive transitions. An example is reported in the inset of Fig.2. It is apparent that the Gaussian approximation does not reproduce at all the shape of the resistive transition. A final note on the 3D-2D dimensional crossover that might be expected at high temperatures, owing to the decrease of the *c*-axis coherence length: being our calculation explicitly 3D, such a crossover is not included there. However, it seems that at least up to  $\sim 120$  K the data are well described by the anisotropic 3D theory. This behavior is completely analogous to the results obtained in dc,<sup>8,15</sup> where a Gaussian analysis of the data sufficiently above  $T_c$  did not show any dimensional crossover, the departure from the AL anisotropic 3D behavior being fully accounted for by the introduction of the cutoff in  $\mathbf{q}$ -space. It would be consistent also with the fact that, on a pure numerical ground, the renormalized coherence length is longer than the bare one, so that the crossover temperature shifts to higher  $T$ , where this phenomenon might be hardly distinguishable from the short-wavelength-fluctuation regime. This point deserves further study in the future.

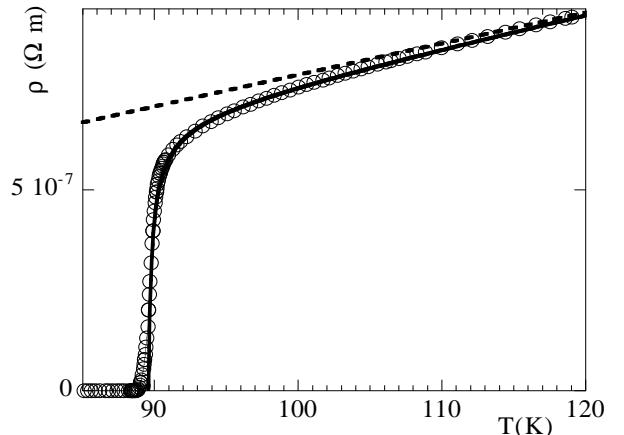


FIG. 4. As in Fig.2, for the dc resistivity on the laser-ablated sample V. For clarity, only 5% of data are shown. The theory reproduces fairly well the shape of the transition above  $T_c$  also in the limit of zero frequency.

In conclusion, we have developed a theory for the finite-frequency fluctuational conductivity in an anisotropic uniaxial superconductor beyond the Gaussian approximation. We have shown that the anisotropy ratio  $\gamma$  en-

ters in a non-trivial way in the expression for the renormalized conductivity. We have performed measurements of the dc and microwave resistivity above  $T_c$  in several YBCO films of different quality and preparation process. In all cases the temperature dependence of the resistivity at all the frequencies investigated could be well described by the theory here developed from slightly above  $T_c$  up to  $\sim 120$  K, with values. Above  $\sim 120$  K the theory does not apply, and more extensions are needed.

## V. ACKNOWLEDGMENTS

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